Population oscillation of the multicomponent spinor Bose-Einstein condensate induced by nonadiabatic transitions

Xiuquan Ma, Lin Xia, Fang Yang, Xiaoji Zhou, Yiqiu Wang, Hong Guo,* and Xuzong Chen†

Key Laboratory for Quantum Information and Measurements, Ministry of Education, School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, Peoples Republic of China

(Received 1 October 2005; published 24 January 2006)

The generation of the population oscillation of the multicomponent spinor Bose-Einstein condensate is demonstrated in this paper. We observe and examine the nonsynchronous decreasing processes of the magnetic fields generated between quadrupole coils and Ioffe coils during the switch-off of the quadrupole-Ioffe-configuration trap, which is considered to induce a nonadiabatic transition. Starting from the two-level Schrödinger equation, we have done some numerical fitting and derived an analytical expression identical to the results of Majorana and Zener, of which both results well match the experimental data.

DOI: 10.1103/PhysRevA.73.013624 PACS number(s): 03.75.Mn, 03.75.Kk, 32.80.Pj

I. INTRODUCTION

Recently, the multicomponent spinor Bose-Einstein condensate (BEC) has become a “hot” topic, since it shows an appealing expectation in providing entangled spin systems applied in quantum optics and quantum computations [1,2], and in the quantized vortex applied in the study of superconductors and superfluidity [3,4]. Since more features [5–9] and more phenomena [10–13] are explored, the quantitatively manipulating and the splitting of the multicomponent spinor BEC are urgently desired. In several groups, the means of optical manipulation has been applied to the splitting and to the study of the spinor BEC [14–17]. However, since the BEC is more commonly generated in static magnetic traps, the splitting and manipulating by the means of a magnetic field will be more convenient and efficient [10].

Nonadiabatic transitions of atoms within magnetic sublevels are very old problems established in about 1930s, to the best of our knowledge, [18–20]. The model of the Majorana transition in which the magnetic field evolves as $B(t)=0$, $B(t)=\text{const}$, $B(t)=kt$ and $t=(-\infty, \infty)$ was created and solved, to the best of our knowledge, by Majorana in 1932 [21]. Then, a more explicit and easily comprehensible explanation about the Majorana transition was described by Rabi [22]. After that, although further theoretical discussions and analyses and other implements such as group theories have emerged to improve the study of the Majorana transition [23–28], few quantitative investigations nor theoretical investigations have been undertaken, because there were not any precise ways to control the fleeting hot atoms, and some qualitative explanations about spin flips and atom loss in magnetic quadrupole traps are well known [29–31]. However, with the development in the experiments and techniques of ultracold atoms, the almost motionless atoms can be provided to interact with the swiftly rotating magnetic field, and this makes the quantitative examination of Majorana transitions possible. In our previous work, we demonstrated that Majorana transitions will emerge during the switch-off of the magnetic trap in our experiment system and induce the split multicomponent spinor BEC [10].

In this paper, we report on the observation and the explanation of the population oscillation of the multicomponent spinor BEC induced by nonadiabatic transitions. We believe that the formation of the population oscillation is caused by the vertical oscillation of the condensate cloud and the nonsynchronous decreasing processes (NDP) of the magnetic fields, which are both measured experimentally and demonstrated in this paper. Further, we derive an analytical expression of this, starting from the Schrödinger equation and the experimental conditions, while the numerical calculation has also been undertaken. Both the analytical and numerical fitting results well fit the experiment data.

II. GENERATING THE POPULATION OSCILLATION OF MULTICOMPONENT SPINOR BEC

Our experiment is set up on a standard equipment system for generating a cigarlike condensate in a dilute gas of $^{87}\text{Rb}$. A compact low-power quadrupole-Ioffe-configuration (QUIC) trap with trapping frequencies of $\omega_r=2\pi\times220\text{ Hz}$ in radial directions and $\omega_z=2\pi\times20\text{ Hz}$ in the axial direction, in which the quadrupole coils are assumed to be along the $x$ direction and the Ioffe coils along the $z$ direction, is mounted on the lower chamber where the vacuum is up to $2\times10^{-11}\text{ mbar}$. After the evaporative cooling, the condensate cloud is held in the center of the magnetic trapping potential generated by $23.6\text{ A}$ current in Ioffe coils and the $24.1\text{ A}$ current in quadrupole coils. In addition, in order to reduce the offset field $B_0$ in the center of the trap, we also apply a couple of compensate coils along the $z$ direction to regulate the tightness of the trap center. When the compensate coils are switched on, the $B_0$ is reduced from...
The vertical down is the direction of the gravity. Different oscillating frequency is due to different tightness of trapping confinement. The oscillation before the time 17.5 ms represents the switch-off of the compensate coils, while after it represents the restoring of the compensate coils. When the compensate coils are switched off, the average velocity of the oscillation is around 1 cm/s, and the period is calculated to be around 11 ms.

First, the compensate coils are switched off, and after a certain time interval, which we can call “coil delay time,” the QUIC trap (the quadrupole coils and Ioffe coils) is switched off afterwards. During this coil delay time, as we know, the condensate cloud will be oscillating up and down vertically due to the balance of gravity and a new trapping potential whose confinement is less tighter than the original one due to the change of $B_0$. To demonstrate the vertical oscillation clearly, we first switch off the compensate coils and then restore it (see Fig. 1). We can see that, at different coil delay time during the oscillation, the condensate cloud may stay at the different altitude which can be expressed as

$$Y_{vertical}(\mu m) = 23.2 \times \cos 0.564(t/\text{ms}) - 28.2,$$

where the positive direction of the $y$ direction is vertically up (opposite to that of gravity). From both the expression Eq. (1) and Fig. 1, we find that the average velocity of the oscillation is around 1 cm/s and the period is calculated to be around 11 ms.

The fact is, although the quadrupole trap and Ioffe trap are switched off at the same time, their decreasing processes are totally nonsynchronous according to our experimental measurement (see Fig. 2). We apply a detective coil to detect the field evolution of the two types of coils, respectively. While one is measured by the detective coil, the other is substituted by an inductance vessel. This means of detection will be separated in space by the Stern-Gerlach effect due to the interaction between atom spins and the gradient of the magnetic field. The population oscillation is shown partly from the expression Eq. 3, and surely a much longer oscillation has been observed in our experiment. Examining all the pictures during the whole process, we find out that the period of the population oscillation is about 11 ms. The total number of the atoms is about $2 \times 10^5$. The most left cloud is considered to represent the $m_F=+2$ component and the most right one belongs to the $m_F=-2$ component, for we assume that the direction of the field after the reversion is the quantum axis. It is necessary to point out that the periods of the vertical oscillations and population oscillation are more or less the same. We believe this means that the population oscillation is induced by the vertical oscillation and the different distributions of atoms are related to the different altitudes of the condensate cloud.

III. THE BASIC MODEL FOR A TWO-LEVEL NONADIABATIC TRANSITION

According to Rabi’s description, the Majorana transitions only take place in the case that the rotating frequency of the magnetic field $f_{Rot} = \omega_B(t)/2\pi\gamma$ is big enough to be comparable to the Larmor frequency of the field $f_{Lar} = g\mu_B\omega_B(t)/2\pi\hbar$ [22]. This is easy to understand from the perspective of magnetic resonant transitions (MRT). Because
the emergence of the zero field and the reversion of the di-
rection often bring the small magnitude of the field and huge
rotating frequency, the nonadiabatic transition will happen
more easily. In our experiment, the evolution of the magnetic
field just coincides with this conclusion see Fig. 4.

When the field reverses its direction, the rotating frequency is re-
markably huge. At this time, the magnetic moment of the
atom cannot follow the rotating field and there will be a
transition among the magnetic sublevels that can be ascribed
to the Majorana transition.

The Majorana formula has been derived from both quan-
tum mechanics and group theories to explain the multilevel
cases [21,28]. For a system with a total angular moment $J$,

$$P_{m,m'} = \frac{(J+m)!(J+m')!(J-m)!(J-m')!(\cos \frac{\theta}{2})^{4J}}{	imes \sum_{\nu=0}^{2J} \frac{(-1)^\nu \tan \frac{\theta}{2}}{\nu!(\nu-m+m')!(\nu-m-m')!(\nu-m')!(\nu-m'-m')!}}^2$$

(2)

where the value of the parameter $\theta$ is given by the two-level transition

$$\sin^2 \frac{\theta}{2} = P_{1/2,-1/2}.$$  

(3)

This means that once we solve the two-level case, the results can be generalized for any system with any value of $J$. So, in this part, we will focus on the physical model of the two-
level nonadiabatic transition.

The time, during which $f_{\text{Rot}}$ is big enough to be compa-
rable to $f_{\text{Lar}}$, is about 1 $\mu$s [see Fig. 4(a)], so the movement of the cloud is about 0.01 $\mu$m according to the average velocity about 1 cm/s. Compared with the vertical oscillation magnitude ($\sim$ 50 $\mu$m), the atom cloud can be seen to be motionless. So we consider a system of motionless atoms.
whose spin moment is \( s = \frac{1}{2} \) with a time evolving magnetic field \( \vec{B}(t) \). In this simple case, we begin with the Schrödinger equation

\[
\frac{i\hbar}{\mathcal{H}} \left( \begin{array}{c} \dot{c}_1 \\ \dot{c}_2 \end{array} \right) = \frac{\mathbf{g} \mu_B}{\hbar} \mathbf{F} \cdot \vec{B}(t) \left( \begin{array}{c} c_1 \\ c_2 \end{array} \right),
\]

where we know \( \mathbf{F} = 2/2\mathcal{H} \) in the two-level case. According to the configuration of the QUIC trap, the center of the magnetic trap is right on the \( z \) axis. In addition to the dragging down of the gravity, however, the center of the total trapping potential is slightly down along the \( y \) direction. So, the three components of the evolving magnetic field should take the form \( \vec{B}(t) = [0, B_y(t), B_z(t)] \). Putting the matrix form into the basic equation Eq. (4) and taking the substitution \( a = g \mu_B / 2\hbar \), one yields

\[
\dot{c}_1 = -ia B_y c_1 - a B_z c_2,
\]

\[
\dot{c}_2 = a B_z c_1 + ia B_y c_2,
\]

and the initial conditions are

\[
c_1(0) = 1,
\]

\[
c_2(0) = 0.
\]

Substituting the real evolution of the magnetic field into Eq. (5), one can get the transition probability.

Though, as we know, the evolution of the magnetic field of the coils is exponentially down due to the discharging process of the coils, i.e.,

\[
B_y(t) = B_{y0} \exp(-\tau_{\theta} t) + B_{yq} \exp(-\tau_q t),
\]

\[
B_z(t) = B_{z0} \exp(-\tau_{\theta} t) - B_{zq} \exp(-\tau_q t),
\]

where \( B_{y0} \) and \( B_{z0} \) are, respectively, the \( y \) direction and the \( z \) direction component of the magnetic field generated from the Ioffe coils, \( B_{yq} \) and \( B_{zq} \) are from the quadrupole coils, and \( \tau_{\theta} \) and \( \tau_q \) are the reciprocals of the exponential time constants \( \tau_{\text{offe}} \) and \( \tau_{\text{quad}} \) in Fig. 2. Since only the transition properties will be taken into account, it is appropriate to take the first-order approximation \( \exp(-\tau_q t) = 1 - \tau_q t \) and describe them in the linear forms [see Fig. 4(b)]

\[
B_y(t) = A_y - C_y t,
\]

\[
B_z(t) = A_z - C_z t,
\]

where

\[
A_y = (B_{y0} + B_{yq}),
\]

\[
A_z = (B_{z0} - B_{zq}),
\]

\[
C_y = (B_{y0} \tau_{\theta} + B_{yq} \tau_q),
\]

\[
C_z = (B_{z0} \tau_{\theta} - B_{zq} \tau_q).
\]

From above we know \( \tau_{\theta} = 1/39 \) ms is twice that of \( \tau_q = 1/91 \) ms (see Fig. 2) and hence \( \tau_q = 2\tau_q \). So the value of \( C_y \) and \( C_z \) can be estimated to be \( C_y = \tau_q (2B_{y0} + B_{yq}) \) and \( C_z = \tau_q (2B_{z0} - B_{zq}) \), and the typical values of theirs satisfy \( C_y > C_z \). So we can simplify the expressions of magnetic fields by fixing the \( B_y \) at the value \( A_y = A_z - C_y t \) at the time \( t = 0 \) when the field \( B \) reverses its direction at \( B_y(t_0) = 0 \),

\[
B_y(t) \approx A_y,
\]

\[
B_z(t) = A_z - C_z t.
\]

With Eq. (10) and all other substitutions taken into Eq. (5), we can get the second-order differential equations which can be transformed into Webber equations [20]. As mentioned above, the transition only occurs when \( B_y \) reverses its direction, so it is allowed to reset the initial condition so as to utilize the asymptotic solutions of Webber equations

\[
c_1(-\infty) = 1,
\]

\[
c_2(-\infty) = 0.
\]

At the same time, the transition probability should take the form \( P = 1 - |c_2(+\infty)|^2 \), since the magnetic field has reversed its direction [20,21].

Therefore, we can derive the analytical expression of the Webber equation with its infinite asymptotic solutions. If we take this substitution

\[
\alpha = \frac{a A_y^2}{2 C_z},
\]

the final result of the analytical expression is (the calculation details can be found in [20,32]):

\[
P_{1/2,-1/2} = \exp(-2\pi \alpha).
\]

The above analytic expression is identical to the results derived previously [20,21]. From this analytical solution, the population oscillation should rely on the magnitude of \( A_y \), which is the only unfixed parameter in \( \alpha \). In our experiment, \( A_{y0} \) is approximately proportional to the altitude oscillation \( Y_{\text{vertical}} \), because the value of \( B_{y0} + B_{yq} \) is around linear in radial directions. Hence, we approximately have \( \alpha = k Y_{\text{vertical}} \) and \( k \) is a constant. Finally, we know that the transition probability will also oscillate with the altitude oscillation \( Y_{\text{vertical}} \)

\[
P_{1/2,-1/2} = \exp(-2\pi k Y_{\text{vertical}}^2),
\]

which confirms that the population oscillation is induced by the vertical oscillation. As mentioned above, the distribution of the atom population among the split multicomponent spinor BEC can be obtained by the two-level result Eq. (12) and the Majorana formula Eqs. (2) and (3).

The numerical fitting of the nonadiabatic transition has also been done by directly taking the exponential expressions of magnetic field \( \vec{B}(t) \) into the basic Schrödinger equation Eqs. (4) or (5). To demonstrate the precision of our analytic analysis, the experiment data, numerical fitting results, and theoretical deducing results are illustrated at the same time (see Fig. 5). Though each sublevel state has its own oscillation pattern, we only illustrate the \( |F = 2, m_F = \mp 2\rangle \) state, for
BEC splitting, which coincides with the explanation of the components. In addition, we analyze the phenomenon of and manipulate the spinor BEC and the distributions of its ment could most probably offer a profound means to produce for its experimental formation. We believe that our experi-

setting the experiment parameters is reported and explained 

ation distribution of the five components can be regulated by 

knowledge, the population oscillation in which the popula-

ated in the magnetic trap. For the first time, according to our 

population oscillation of the five components of $^{87}\text{Rb}$ gener-

15 to 40 ms. 

instance. Since the experimental points before 15 ms of the coil delay time are slightly affected by the eddy current of coils and mountings, we merely count the data ranging from 15 to 40 ms.

IV. CONCLUSION

To conclude, we observed the split spinor BEC and the population oscillation of the five components of $^{87}\text{Rb}$ generated in the magnetic trap. For the first time, according to our knowledge, the population oscillation in which the population distribution of the five components can be regulated by setting the experiment parameters is reported and explained for its experimental formation. We believe that our experiment could most probably offer a profound means to produce and manipulate the spinor BEC and the distributions of its components. In addition, we analyze the phenomenon of BEC splitting, which coincides with the explanation of the NDP proposed in Reference 33, and measure the value of the DNP experimentally.

Starting with the Schrödinger equation, together with the experimental conditions, we derive the analytical solution of the nonadiabatic Majorana transition of condensate atoms within the magnetic sublevels. Although the Majorana transition has been broadly applied in cold atom physics for qualitative explanations, it is for the first time, to the best of our knowledge, that we quantitatively apply it to well fit our experimental data of the nonadiabatic transition.

For further experiments, we have designed a more complex system to control the magnetic fields of the two types of coils separately, and with the aids of the theoretical analysis mentioned above, the arbitrary manipulations of the atom populations among the multicomponent spinor BEC are anticipated. Besides, we also plan to load the separated multi-component condensates into the optical trap and study the interaction between the different components. To summarize, our experimental method provides a convincing and potential way of studying the ultracold atom physics, including BEC and atom optics.

ACKNOWLEDGMENTS

We thank Dr. Shuai Chen for his wonderful previous work on our experiment system and helpful discussions with us. We thank Professor Jörg Schmiedmayer and Dr. Stephan Schneider for their helpful suggestion in our experiment. We thank Mr. Lin Yi for his technical supports on LabVIEW programming work. This work was supported by the National Fundamental Research Programme of China under Grant Nos. 2001CB309308, and 2005CB3724503, the Major Program of National Natural Science Foundation of China under Grant No. 60490280, National Natural Science Foundation of China under Grant Nos. 60271003 and 10474004, the National High Technology Research and Development Program of China (863 Program), the International Cooperation Program under Grant No. 2004AA1Z1220, and partially by the DAAD Exchange Program (Program NO. D/0212785 Personenaustausch VR China), and the DAAD exchange program: Program NO. D/05/06972 Projektsbezogener Personenaustausch mit China (Germany/China Joint Research Program).