




RESEARCH ARTICLE | SEPTEMBER 24 2020

A linewidth locking method to control the microwave power in optically pumped cesium-beam clocks

Weibin Xie ; Qing Wang ; Xuan He; Zezheng Xiong; Nan Chen; Shengwei Fang; Xianghui Qi; Xuzong Chen 

 Check for updates

Rev. Sci. Instrum. 91, 094708 (2020)

<https://doi.org/10.1063/1.5144478>


View Online


Export Citation

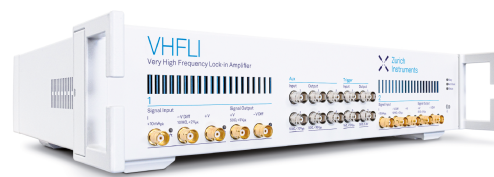
Articles You May Be Interested In

Frequency instability of a miniature optically pumped cesium-beam atomic frequency standard

Rev. Sci. Instrum. (July 2020)

Space qualified microwave source for cold atom clock operating in orbit

Rev. Sci. Instrum. (November 2018)



 Zurich Instruments

Freedom to Innovate.

The New VHFLL 200 MHz Lock-in Amplifier.

Orchestrate pulses, triggers, and acquisition as the hub of your experiment. Discover more – run every signal analysis tool, simultaneously.

Order now

A linewidth locking method to control the microwave power in optically pumped cesium-beam clocks

Cite as: Rev. Sci. Instrum. 91, 094708 (2020); doi: 10.1063/1.5144478

Submitted: 28 December 2019 • Accepted: 8 September 2020 •

Published Online: 24 September 2020



View Online



Export Citation



CrossMark

Weibin Xie,^{a)}  Qing Wang,^{b)}  Xuan He, Zezheng Xiong, Nan Chen, Shengwei Fang, Xianghui Qi, and Xuzong Chen^{c)} 

AFFILIATIONS

Institute of Quantum Electronics, School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, China

^{a)} Author to whom correspondence should be addressed: weibinxie@pku.edu.cn

^{b)} Electronic mail: qw1988@pku.edu.cn

^{c)} Electronic mail: xuzongchen@pku.edu.cn

ABSTRACT

In this paper, we present a linewidth locking method to control the microwave power in optically pumped cesium-beam frequency standards. The responses of optically pumped cesium-beam tubes and classical cesium-beam tubes are analyzed and compared against the power of the microwave field. Due to the wide probability distribution of atomic velocity resulting from the optical state preparation and detection, the linewidth of the Ramsey pattern is sensitive to the microwave power. The results can be used to control the microwave power instead of using the traditional extremum method. The advantages of the new method are discussed, and we named this new method the linewidth locking method. When the microwave power is well controlled at a low level by the linewidth locking method, the frequency stability of cesium-beam clocks will be improved to a certain degree for the reduction of the Ramsey pattern linewidth. In experiment, using the linewidth locking method, the Allan deviation of our optically pumped cesium-beam frequency standard is $2.64 \times 10^{-12}/\sqrt{\tau}$ and continues until the averaging time exceeds 1×10^5 s, which is 17% better than that using the traditional extremum method.

Published under license by AIP Publishing. <https://doi.org/10.1063/1.5144478>

I. INTRODUCTION

To date, the cesium-beam frequency standard is still unique and irreplaceable in the field of time-keeping systems owing to the excellent performance of the long-term frequency stability.^{1,2} Studies pointed out that the drift of the microwave power had an important influence on the long-term stability of the cesium-beam frequency standard.^{3–6} For the servo control system of the microwave power, the servo signal must be directly derived from the quantum system in the cesium beam, to prevent influences from the outside environment. The extremum method⁶ provides a solution to the problem, making the classical cesium-beam clock, in which state selection is accomplished through a deflection magnet, and the method was successful in the early 1990s. For further improvements after that, optical pumping has been widely used as a replacement of magnetic state-selection by many research groups.^{7–10} In this paper, a linewidth locking method is proposed for the first time

to control the microwave power in optically pumped cesium-beam (OPCB) frequency standards instead of the traditional extremum method, which will simplify the design of servo control systems and contribute to the frequency stability of cesium-beam frequency standards.

II. EXPERIMENTAL SETUP

Figure 1 shows a schematic of the miniature optically pumped cesium-beam frequency standard, which is the subject of this study, including a cesium oven, optical pumping and detection regions, microwave excitation regions, a C-field coil (the box around the microwave cavity), an optical system, and an electronic system. Apart from the optical system and the electronic system, all components are sealed in a vacuum tube covered by three layers of magnetic shielding material. This is often called a cesium-beam tube.

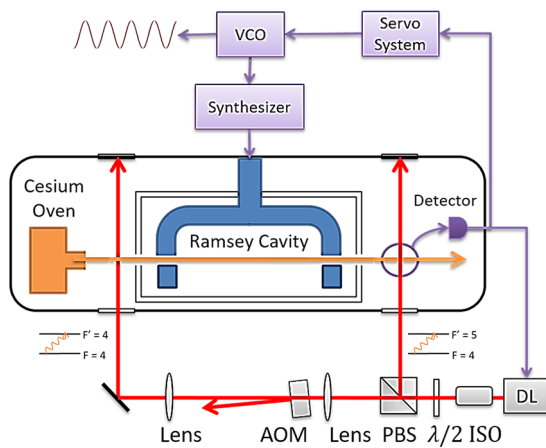


FIG. 1. Schematic of the miniature optically pumped cesium-beam frequency standard. Reprinted with permission from Q. Wang, "Research on miniature optically pumped cesium beam frequency standard," Ph.D. thesis, Peking University, 2014.¹¹

The $F = 4 \rightarrow F' = 4$ transition of the Cs D_2 line is used for optical pumping, and the $F = 4 \rightarrow F' = 5$ transition is used for detection (Fig. 2). The radio-frequency interrogation is classically performed in a Ramsey microwave cavity, but first, the atoms have to be

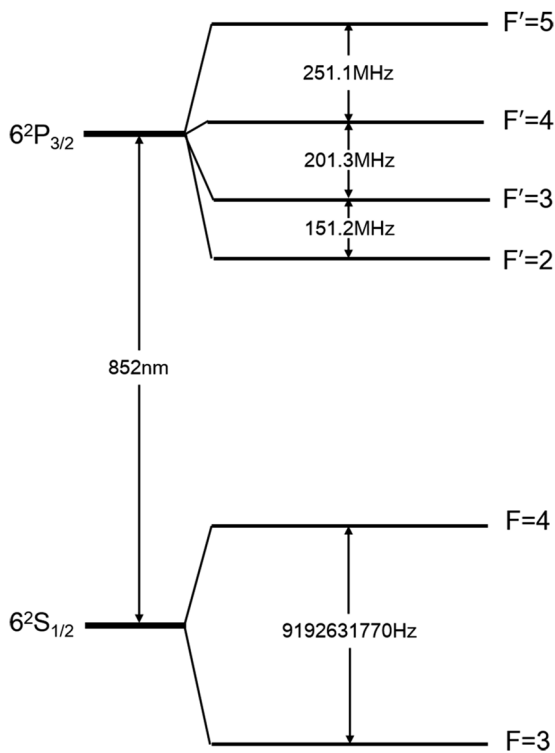


FIG. 2. Energy levels of the Cs D_2 line.

prepared in a single hyperfine atomic state $F = 3$. This preparation is performed by optical pumping instead of magnetic state selection. In this case, there is no spatial separation, and all the atoms pass through the laser toward the microwave cavity. Finally, detection of cesium atoms is performed using the standard optical technique in the detection region, and the fluorescence emitted by the atoms upon decay to the ground state is used as a measure of the population in state $F = 4$ and is a measure of the number of atoms that have made a transition in the Ramsey interaction region. The fluorescence photons of these atoms are detected by a photodiode.

III. VELOCITY DISTRIBUTION THEORY

For an E-plane-type Ramsey cavity with an interaction region length l and a drift region length L , the Ramsey transition probability can be represented by the following expression:¹²

$$P(b, \Delta, T) = \frac{4b^2}{\Omega^2} \sin^2 \frac{\Omega t}{2} \left(\cos \frac{\Omega t}{2} \cos \frac{\Delta T}{2} - \frac{\Delta}{\Omega} \sin \frac{\Omega t}{2} \sin \frac{\Delta T}{2} \right)^2. \quad (1)$$

Here, the quantity b represents the amplitude of the microwave interrogation field, $\Delta = \omega - \omega_0$ is the detuning between the microwave frequency ω and the atomic resonance frequency ω_0 , and $T = L/v$ and $t = l/v$ are the transit times for an atom of velocity v propagating through the drift and interaction regions, respectively. $\Omega = \sqrt{b^2 + \Delta^2}$ represents the generalized Rabi frequency.

In fact, the final Ramsey lineshape is a weighted average of the atomic velocity distribution,

$$P(b, \Delta) = \int_0^\infty P(b, \Delta, T) \rho(T) dT, \quad (2)$$

where $\rho(T)$ is the distribution of the transit time T , which can be deduced from the atomic velocity distribution directly.

In OPCB tubes, $\rho(T)$ can be expressed as follows:¹³

$$\rho_n(T) = \frac{C_n}{T_0} \left(\frac{T_0}{T} \right)^{n+2} \exp(-T_0^2/T^2). \quad (3)$$

Here, $T_0 = L/\alpha$, and $\alpha = \sqrt{2k_b (T_{emoven} + 273)/M}$ is the most probable velocity in the cesium oven, where k_b is the Boltzmann constant, M is the cesium atom mass, T_{emoven} is the temperature of the cesium oven, and C_n is the normalization constant. The case $n = 3$ with $C_3 = 2$ describes the transit time distribution of the beam at the exit of the cesium oven, and it is the simplest case in which a single laser is used for the atomic state preparation and detection of the transition by optical pumping (Fig. 3). The optical transition involved is saturated for most of the interaction times for the light beam power densities actually achievable. One may thus assume that the population difference does not depend on the velocity.¹³ If more than one laser line is available, the value of n may be 1 or 2, with $C_1 = 2$ and $C_2 = 4/\sqrt{\pi}$, depending on the effects of velocity selection created by the nature of the optical pumping and detection.¹³⁻¹⁵

For the case of $n = 2$, when a light source with a different wavelength is available for detection (Fig. 1), a cycling two-level transition, the $F = 4 \rightarrow F' = 5$ transition (Fig. 2), may be chosen, which yields a number of fluorescence photons proportional to the interaction time, thus, introducing a weighting factor of T .

For the case of $n = 1$, on the basis of $n = 2$, a two-wavelength light beam is used for optical pumping (Fig. 4) to transfer the

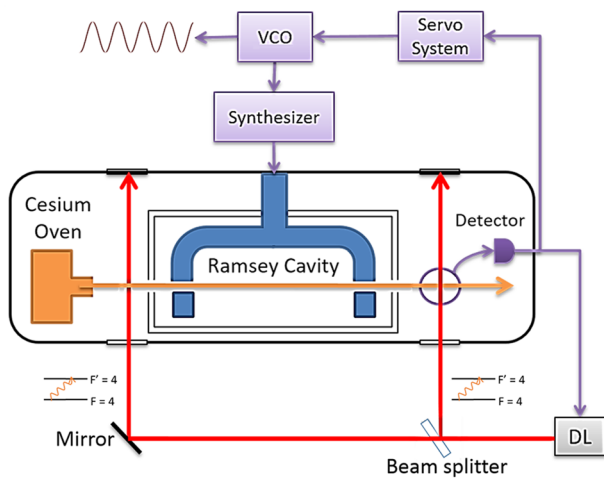


FIG. 3. Optically pumped cesium-beam frequency standard in which a single laser is used.

population of the 16 hyperfine sublevels as efficiently as possible to a single $m_F = 0$ level.¹⁶ However, the saturation of this two-photon process requires a larger energy density or a longer interaction time, which may not be achieved in practice.¹³ As a first approximation, the population difference is proportional to the interaction time with the light beam, which introduces a weighting factor of T .

In the OPCB tube, the state preparation is performed by optical pumping instead of magnetic state selection. In this case, there is no spatial separation, and all the atoms pass through the laser toward the microwave cavity and detection region, so the distribution of

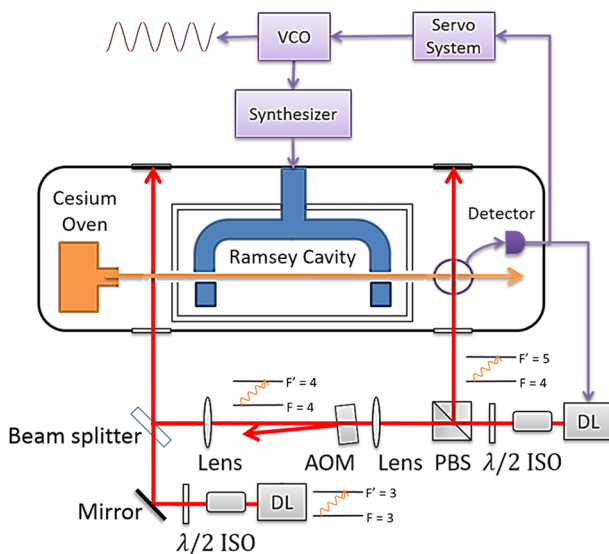


FIG. 4. Optically pumped cesium-beam frequency standard in which a two-wavelength light beam is used for optical pumping and a cycling two-level transition is used for optical detection.

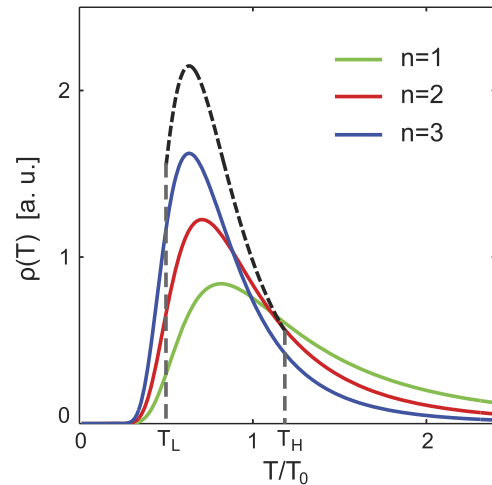


FIG. 5. Transit time distribution functions: three solid lines refer to optical pumping and detection in the case of $n = 1, 2,$ and $3,$ respectively; the black dashed line refers to magnetic state selection in the case of the truncated Maxwell distribution, where T_L and T_H are the low and high cut-offs of the transit time, respectively. Here, $T_0 = L/\alpha,$ and α is the most probable velocity of atoms.

velocity is complete. In contrast to the OPCB tube, in the classical magnetic state selection cesium beam (MSCB) tube, the deflection of the trajectories depends on the atom velocity, atoms which are too slow or too fast do not strike the hot-wire detector, and consequently, $\rho(T)$ can be written as a truncated Maxwell distribution,¹³ with T_L and T_H representing the low and high cut-offs of the transit time, respectively,

$$\rho(T) = \frac{C_0}{T_0} \left(\frac{T_0}{T}\right)^5 \exp(-T_0^2/T^2) \quad (T_L \leq T \leq T_H). \quad (4)$$

Figure 5 shows these transit time distribution functions. All calculations and figures in the paper are based on parameters of the OPCB frequency standard developed in our laboratory, such as $l = 1.0$ cm, $L = 16.7$ cm, and $Tem_{oven} = 100$ °C.

IV. SERVO CONTROL OF THE MICROWAVE POWER

The servo system cyclically locks the microwave frequency, microwave power, and C field. The time for each of these three loops to complete one lock is 1 s. The time sequences of the three loops are as follows: the frequency servo works continuously for 100 s, then the power servo works for 1 s, then the frequency servo works for 100 s, then the C field servo works for 1 s, and these four processes are repeated.

Because the microwave power is proportional to the value of b^2 , we define the relative microwave power W_r as

$$W_r = 10 \log_{10}(W/W_{opt}) = 20 \log_{10}(b/b_{opt}). \quad (5)$$

Here, W is the microwave power, and b_{opt} is the optimum value of the microwave interrogation field, which yields the largest maximum value $P(b_{opt}, \Delta = 0)$ of the Ramsey transition probability.¹³

Figure 6 shows the cesium-beam tube responses $P(b, \Delta)$ vs the microwave power at $\Delta = 0$ for different transit time distribution

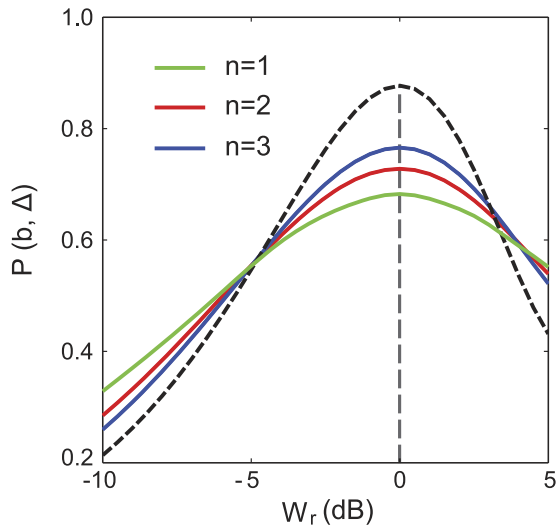


FIG. 6. Variation in the Ramsey probability $P(b, \Delta)$ vs the relative microwave power W_r for different values of n . The dashed line refers to the case of magnetic state selection. When b is adjusted at its optimum value b_{opt} , W_r equals zero, and the response curve of the beam tube reaches its maximum. With the decrease in n , the response curve tends to be flat, and the transit time distribution tends to be wide. It is assumed that $\Delta = 0$.

functions, and it is clear that every curve presents a peak structure. For the convenience of comparison, the peak of each curve is aligned at the origin of the horizontal axis, where b is equal to b_{opt} . The traditional extremum method achieves locking the microwave power to the optimum value by applying a square wave modulation to the microwave amplitude around the peak of the response curve.^{6,14}

However, combining Figs. 5 and 6, we can see that the response curve in the OPCB tube is flatter at the top than that in the MSCB tube because of the wider transit time distribution.¹³ It means that the Q value of the reference curve is low, which limits the servo accuracy of the microwave power in the OPCB clock.

Here, we propose another approach to accomplish the servo control system of the microwave power in OPCB atomic clocks. In fact, owing to the wide distribution of transit time, the Ramsey linewidth, usually called the full width at half maximum (*FWHM*), depends heavily on the microwave power in OPCB tubes. Such dependence is not obvious in the cesium-beam clock with magnetic state selection. Figure 7 shows the dependence on the tubes of OPCB (three solid lines) and of MSCB (dashed line), obtained by numerical calculation. So the servo control of microwave power can be achieved by maintaining a constant Ramsey *FWHM*: First, set an appropriate initial value of the Ramsey *FWHM*, and then periodically collect the current Ramsey *FWHM* and compare it with the initial value to obtain error signals, then feed the error signals back to the control system of the microwave power.

There are many frequency shifts coupling microwave power instability to the clock's frequency variations, such as Rabi pulling, cavity pulling, and second-order Doppler effect.^{17,18} It is reported that the sensitivity of the clock's frequency stability to microwave

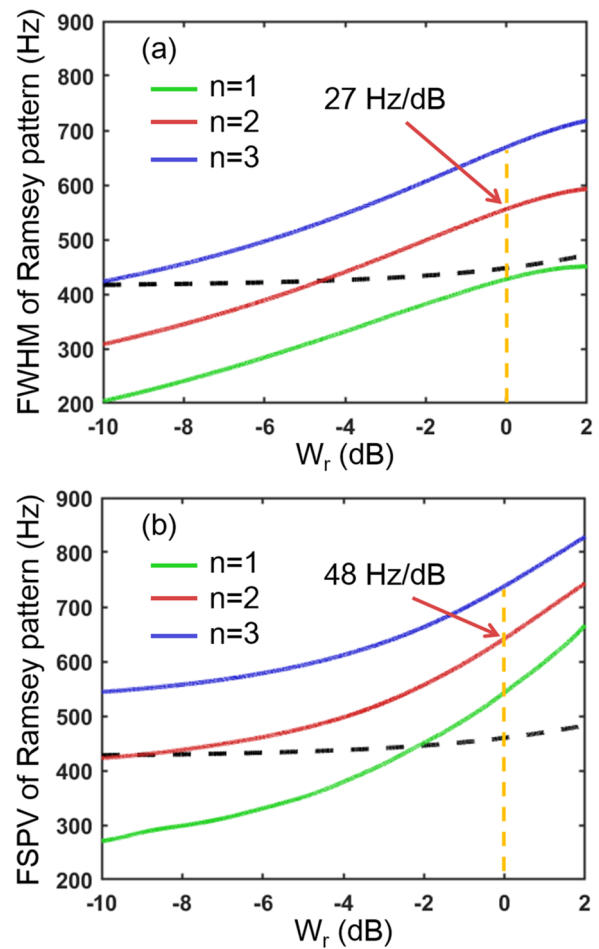


FIG. 7. Variation in the Ramsey pattern linewidth against the relative microwave power W_r for given transit time distribution functions. The dashed line corresponds to the truncated Maxwell distribution with $T_L = T_0$ and $T_H = 2T_0$ assumed. The linewidths of the Ramsey pattern are generally more sensitive to the microwave power in OPCB tubes than that in MSCB tubes, for given transit time distribution functions. (a) *FWHM* is full width at half maximum and (b) *FSPV* is frequency separation between the peak and the valley of the Ramsey pattern.

power is about 10^{-13} to 10^{-15} dB^{-1} .⁴ For the OPCB frequency standard with $n = 2$ in our laboratory, when b is around b_{opt} , the microwave power sensitivity is measured to be about 5×10^{-13} dB^{-1} , while the sensitivity of the Ramsey *FWHM* to the microwave power is measured to be ~ 28 Hz/dB , which is close to the theoretical result shown in Fig. 7(a). Therefore, according to a simple calculation, the effect of microwave power drift on the long-term stability of the atomic clock can be reduced to a low level of 5×10^{-15} , as long as the *FWHM* of the Ramsey pattern is stabilized with less than about 0.28 Hz residual fluctuation.

However, it is time consuming to obtain the *FWHM* of the Ramsey pattern precisely by circuits and programs. Fortunately, as is shown in Figs. 8 and 7(b), the frequency separation between the peak and the valley (*FSPV*) of the Ramsey pattern shares similar

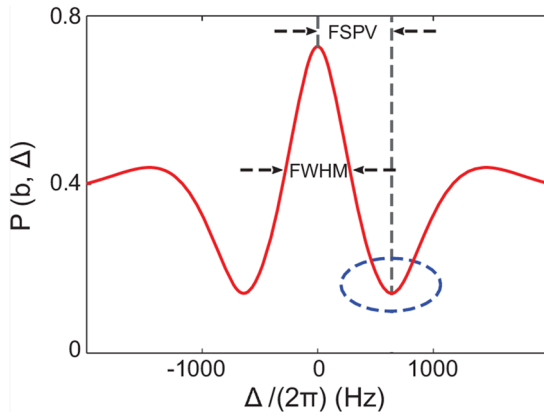


FIG. 8. Frequency separation between the peak and the valley (*FSPV*) of the Ramsey pattern. The frequency corresponding to the peak, which served as the servo reference for the clock frequency, is very stable, and the *FSPV* is only determined by the position of the valley. It is assumed that $b = b_{\text{opt}}$, with $n = 2$.

properties with the *FWHM*. Hence, we can take *FSPV* as an appropriate alternative, for simplicity of realization. The peak of the Ramsey pattern is corresponding to the reference line of the clock frequency, which is very stable. It means that the servo control of the microwave power is converted into the problem of accurately locking the valley position f_v of the Ramsey pattern (Fig. 8), which can be simply realized by applying a modulation to the microwave frequency around the valley.

In summary, the traditional extremum method achieves locking the microwave power to the optimum value by locking the microwave amplitude around the peak of the response curve; because of the sharper response curve with a higher *Q* value in the MSCB tube than that in the OPCB tube, the traditional method is better used for the MSCB tube. Because of the higher sensitivity of the Ramsey *FSPV* to the microwave power in the OPCB tube than that in MSCB tube, the linewidth locking method is better used for the OPCB tube.

V. DISCUSSION

A. Effects of the cesium oven temperature

No matter which method is used to control the microwave power in OPCB frequency standards, the temperature Tem_{oven} of the cesium oven should be considered, which is the only factor affecting the servo accuracy.

As to the traditional extremum method, the only servo reference is W_{opt} corresponding to b_{opt} , which varies with Tem_{oven} slightly. In order to show the sensitivity of the servo reference to Tem_{oven} , we have calculated the variation in $W_{\text{opt}}(Tem_{\text{oven}})$ relative to the case of $Tem_{\text{oven}} = 100^\circ\text{C}$, as is shown in Fig. 9(a),

$$\begin{aligned} S_{b_{\text{opt}}} &= 10 \log_{10} \left(\frac{W_{\text{opt}}(Tem_{\text{oven}})}{W_{\text{opt}}(100^\circ\text{C})} \right) \\ &= 20 \log_{10} \left(\frac{b_{\text{opt}}(Tem_{\text{oven}})}{b_{\text{opt}}(100^\circ\text{C})} \right). \end{aligned} \quad (6)$$

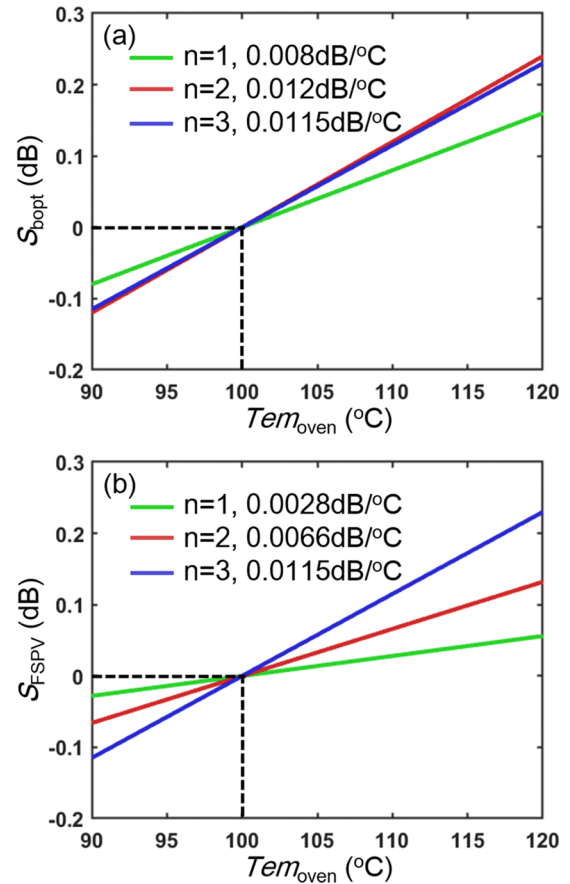


FIG. 9. Variations in the servo reference for controlling the microwave power in the OPCB tube vs the cesium oven temperature for given transit time distribution functions. It is assumed that the microwave power is stabilized to W_{opt} by (a) using the traditional extremum method and (b) using the linewidth locking method.

When using the linewidth locking method to stabilize the microwave power to W_{opt} , the servo reference is the *FSPV*, which also changes with Tem_{oven} slightly (Fig. 10). In order to show the sensitivity of the servo reference to Tem_{oven} , we have calculated the variation in $F_{\text{SPV}}(Tem_{\text{oven}})$ relative to the case of $Tem_{\text{oven}} = 100^\circ\text{C}$,

$$F_r(Tem_{\text{oven}}) = F_{\text{SPV}}(Tem_{\text{oven}}) - F_{\text{SPV}}(100^\circ\text{C}). \quad (7)$$

For comparison, F_r should be converted into the form of the microwave power,

$$S_{F_{\text{SPV}}}(Tem_{\text{oven}}) = F_r(Tem_{\text{oven}})/k. \quad (8)$$

Here, $k = \delta_{F_{\text{SPV}}}/\delta_{W_r}$ is the slope of the curve, as shown in Fig. 7(b), at $W_r = 0$ with $Tem_{\text{oven}} = 100^\circ\text{C}$. Within the scope of this article, Tem_{oven} varies from 90°C to 120°C . According to Fig. 9(a), the corresponding maximum variation in microwave power is from -0.12 dB to 0.24 dB. In addition, according to Fig. 7(b), the slopes of the curves hardly change in this range, so Eq. (8) is valid when Tem_{oven} varies from 90°C to 120°C . Results were obtained by numerical calculation, as shown in Fig. 9(b), where it can be noticed that the

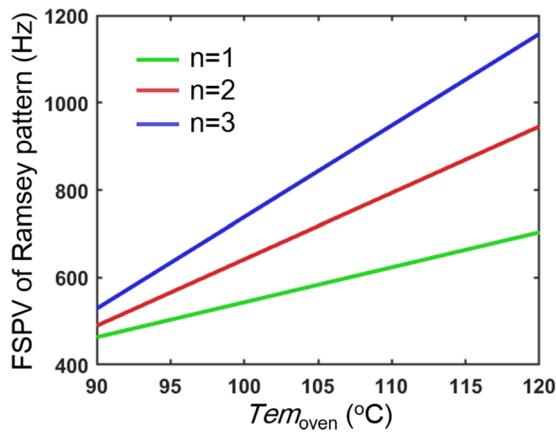


FIG. 10. Relationship between FSPV and the oven temperature, obtained by numerical calculation.

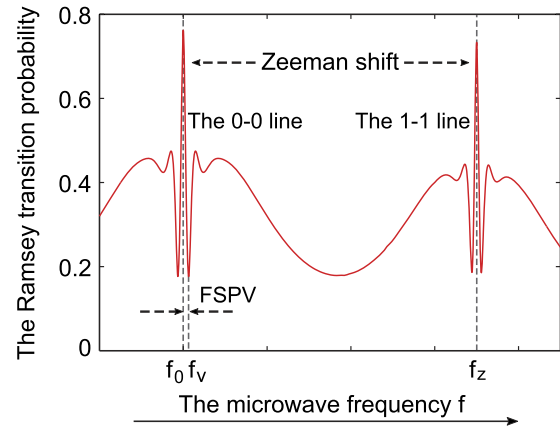


FIG. 11. Fractional microwave spectrum of a cesium-beam frequency standard: f_z is the center of the 1-1 line. The FSPV is used for the servo control of the microwave power, and the Zeeman shift is used for servo control of the C field.

servo references are more insensitive to the cesium oven temperature by using the linewidth locking method than using the extremum method, for given values of $n = 1$ and 2 . In the case of $n = 3$, the two methods bring about the same sensitivity to the oven temperature.

For the sensitivity of our OPCB frequency standard's frequency to the oven temperature, it is too small to be measured under the conditions we have, so we just make an estimate. For the OPCB frequency standard with $n = 2$ in our laboratory, when b is around b_{opt} , the microwave power sensitivity is measured to be about $5 \times 10^{-13} \text{ dB}^{-1}$, and according to the sensitivity of b_{opt} to Tem_{oven} (Fig. 9), we estimate that the sensitivity of the clock's frequency to the oven temperature will not exceed $6 \times 10^{-15} \text{ }^\circ\text{C}^{-1}$. If the linewidth locking method is applied, the sensitivity of the clock's frequency to the oven temperature will be about $3.3 \times 10^{-15} \text{ }^\circ\text{C}^{-1}$.

B. Simplification of circuits designs

Compared with the amplitude modulation (AM) of the classical extremum method, the linewidth locking method is based on frequency modulation (FM) instead. By using direct digital frequency synthesis (DDS), it is convenient to implement the modulation depth and modulation frequency precisely and flexibly.

Most of all, in the circuit design, we can combine the two servo control systems of the microwave power and the C field. As another important source contributing to the fractional uncertainty of the clock frequency,⁴ the C field is commonly stabilized by locking the Zeeman shift (Fig. 11),¹⁹ which is the frequency separation between the center of the ($F = 4, m_F = 0$) \leftrightarrow ($F = 3, m_F = 0$) transition line (0-0 line) and that of any adjacent Ramsey pattern [the ($4, 1$) \leftrightarrow ($3, 1$) transition line or the ($4, -1$) \leftrightarrow ($3, -1$) transition line]. More specifically, using the linewidth locking method makes the servo control systems of microwave power and the C field share the same technical means and conduces to simplify the circuit design.

C. Improvement in the frequency stability performance

In contrast to a single value of W_{opt} as the servo reference provided by the extremum method, we can arbitrarily set the target

value of microwave power within a certain range with the help of the linewidth locking method. In the following sections, we will demonstrate that the frequency stability of the atomic clock will be improved to a certain degree, if the value of the microwave interrogation field is less than b_{opt} .²⁰

For the OPCB frequency standard with $n = 2$ in our laboratory, the FSPV of the Ramsey pattern is used to control the microwave power in experiment. In order to improve the frequency stability while maintaining the sensitivity of FSPV to microwave power, the value of microwave power is 3 dB lower than the optimum value. Figure 12 shows the Allan deviations of our OPCB frequency standard with different methods for controlling microwave power.

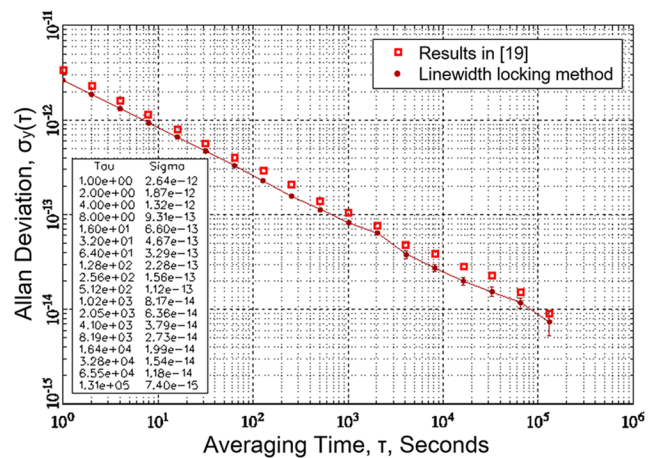


FIG. 12. Allan deviations of the optically pumped cesium-beam frequency standard with $n = 2$ in our laboratory against the reference hydrogen maser. Red-solid circles are the Allan deviation by using the linewidth locking method to control the microwave power 3 dB below the optimal value. Red-hollow squares are the results by using the traditional extremum method to control the microwave power at the optimal value.¹⁹

The Allan deviations are measured against the reference hydrogen maser (VCH-1008, VREMYA-CH). A picosecond resolution instrument (PicoTime-1U, Orolia) was used to perform the frequency comparison. Using the traditional extremum method to lock the microwave power at the optimal value, the Allan deviation is $3.12 \times 10^{-12}/\sqrt{\tau}$,¹⁹ which is almost the same as that with the linewidth locking method at the same microwave power. When using the linewidth locking method to lock the microwave power 3 dB lower than the optimum value, the Allan deviation is $2.64 \times 10^{-12}/\sqrt{\tau}$, which shows a 17% improvement and continues until the averaging time exceeds 1×10^5 s.

VI. CONCLUSION

In conclusion, we have described a novel linewidth locking method for optically pumping cesium-beam clocks to stabilize the microwave power. This method has been shown to be able to provide arbitrary servo reference within a certain range around or below the optimum value of the microwave interrogation field due to the wide distribution of atomic velocity. In addition, the servo reference is insensitive to the drift of the cesium oven temperature. In addition, the mode of frequency modulation enables us to unify the servo systems of the C field and microwave power while simplifying the relevant design of circuits. Finally, by using the new method, it is convenient to accurately stabilize the microwave power at a relatively low level, where the frequency stability of the atomic clock is prospective to be improved by certain degrees owing to a relatively narrow Ramsey linewidth. In the experiment, the frequency stability of the OPCB frequency standard has been improved by 17% to $2.64 \times 10^{-12}/\sqrt{\tau}$ and continues until the averaging time exceeds 1×10^5 s, which proves the advantage of the linewidth locking method.

ACKNOWLEDGMENTS

This work was supported in part by the National Key Research and Development Program of China, under Grant No. 2016YFA0301501, in part by the National Natural Science Foundation of China, under Grant Nos. 61475007, 11334001, and 91336103, and in part by the China Postdoctoral Science Foundation, under Grant No. 2016M590018.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

- 1 J. A. Kusters, L. S. Cutler, and E. D. Powers, in *1999 Joint Meeting EFTF-IEEE IFCS* (IEEE, Besancon, 1999), Vol. 2, pp. 159–163.
- 2 T. E. Parker and J. Levine, *IEEE Trans. Ultrason., Ferroelectr., Freq. Control* **44**, 1239 (1997).
- 3 A. D. Marchi, *IEEE Trans. Ultrason., Ferroelectr., Freq. Control* **34**, 598 (1987).
- 4 A. D. Marchi, in *41st Annual Frequency Control Symposium* (IEEE, Philadelphia, 1987), pp. 53–58.
- 5 C. Audoin, N. Dimarcq, V. Giodano, and J. Viennet, *IEEE Trans. Ultrason., Ferroelectr., Freq. Control* **39**, 412 (1992).
- 6 L. S. Cutler and R. P. Giffard, in *Proceedings of IEEE Frequency Control Symposium* (IEEE, Hershey, 1992), pp. 127–133.
- 7 T. McClelland, I. Pascaru, J. Zacharski, N. H. Tran, and M. Meirs, in *41st Annual Frequency Control Symposium* (IEEE, Philadelphia, 1987), pp. 59–65.
- 8 P. Petit, V. Giordano, B. Bousset, C. A. P. Cerez, G. Theobald, and N. Dimarcq, in *Proceedings of Conference on Precision Electromagnetic Measurements Digest* (IEEE, Boulder, 1994), p. 133.
- 9 B. Bousset, G. L. Leclin, F. Hamouda, P. Cerez, and G. Theobald, *IEEE Trans. Ultrason., Ferroelectr., Freq. Control* **46**, 366 (1999).
- 10 R. Lutwak, D. Emmons, R. M. Garvey, and P. Vlitias, in *33rd Annual Precise Time and Time Interval (PTTI) Meeting*, 2001.
- 11 Q. Wang, “Research on miniature optically pumped cesium beam frequency standard,” Ph.D. thesis, Peking University, 2014.
- 12 N. F. Ramsey, *Phys. Rev.* **78**, 695 (1950).
- 13 J. Vanier and C. Audoin, *The Quantum Physics of Atomic Frequency Standards* (IOP Publishing, Philadelphia, 1989).
- 14 C. Audoin, F. Hamouda, L. Chassagne, and R. Barillet, *IEEE Trans. Ultrason., Ferroelectr., Freq. Control* **46**, 407 (1999).
- 15 P. Cerez, G. Avila, E. de Clercq, M. de Labachellerie, and M. Tetu, in *38th Annual Frequency Control Symposium* (IEEE, Philadelphia, 1984), pp. 452–457.
- 16 G. Avila, V. Giordano, V. Candelier, E. de Clercq, G. Theobald, and P. Cerez, *Phys. Rev. A* **36**, 3719 (1987).
- 17 H. S. Lee, T. Y. Kwon, H.-S. Kang, Y.-H. Park, S. E. Park, H. Cho, and V. G. Minogin, *Metrologia* **40**, 224 (2003).
- 18 S. Guerandel, E. de Clercq, R. Barillet, and C. Audoin, in *IEEE International Frequency Control Symposium* (IEEE, 2007), pp. 1050–1055.
- 19 W. B. Xie, Q. Wang, X. He, N. Chen, Z. Z. Xiong, S. W. Fang, X. H. Qi, and X. Z. Chen, *Rev. Sci. Instrum.* **91**, 074705 (2020).
- 20 J. H. Shirley, W. D. Lee, and R. E. Drullinger, *Metrologia* **38**, 427 (2001).